Pre-filter to robustify the exact linearization based tracking controller of a SCARA type robot[1]

Na Wang, Bálint Kiss
Budapest University of Technology and Economics Budapest, Hungary

Email:{wangna,bkiss}@iit.bme.hu

Abstract. Robustness against parameter uncertainties is an important requirement for robot controllers to achieve high precision and fast tracking. This paper proposes a procedure to cover the uncertainties remaining after the exact linearization (also known as the computed torque method in robotics), and to design am additional linear compensator $K(s)$ to ensure robust performance and stability. The design of $K(s)$ involves standard $\mathcal{H}_\infty$. The procedure is presented in details for a model of a four degree-of-freedom SCARA manipulator. The load mass and friction coefficients are considered as uncertain parameters. The simulation shows that the proposed method can increase robustness against parameter uncertainty.

1. Introduction

The number of industrial robots is continuously increasing. Their precision and speed determine the quality and efficiency of many manufacturing processes. This is also true for SCARA type (Selective Compliance Assembly Robot Arm) devices which are widely used in the electronics industry for assembly and pick-and-place tasks.

Robot arms are nonlinear, MIMO dynamical systems. Tracking precision and speed are conflicting requirements since controllers need to deal with model uncertainties whose effects increase with speed where coupling effects are more important. Manipulator segment and load inertia, friction coefficients, torque constants of the actuators are known with limited accuracy. Various methods have been already explored for robot controller design and implementation including sliding mode control [1],[2], neural networks [3],[4], fuzzy logic [5],[6], and neuro-fuzzy [7],[8], to name a few.

The so-called computed torque method is also known in the literature. The possible drawback of the method, also referred to as exact linearization, is its sensibility to parameter uncertainty. This paper suggests a novel method to cope with parameter uncertainties while applying the computed torque method. The method is based on the design of a pre-filter for a set of uncertain dynamics. The set is determined by taking the combination of the nonlinear robot dynamics for different parameter values in their uncertainty range with the linearizing feedback which works with the nominal parameter values. By linearizing this set of closed-loop dynamics one gets a set of linear systems which is then covered by an output multiplicative uncertainty structure for which a pre-filter is designed using $\mathcal{H}_\infty$ techniques.

The design steps to obtain the pre-filter are presented in Section 2. The suggested method is applied to a SCARA type robot in Section 3.

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2. The robustifying pre-filter and its design

Let us assume that there are N uncertain parameters $(p_1, p_2, \dots, p_N)$ such that each parameter may take its value from a bounded set, i.e $p_i \in Q_i$ and $Q = Q_1 \times Q_2 \dots \times Q_N$. Consider a nonlinear dynamical system with an n-dimensional state vector x such that the state equation reads $\dot{x} = f(x, u, p)$ and the output equation is $y = h(x, p)$ where the dimension m of the input vector u equals the dimension of the output vector y. Suppose that an eventually dynamic linearizing feedback can be calculated for all possible parameter values in Q. Since the real values of the parameters are unknown, the feedback is applied for some nominal values p^0. The feedback law is given as $\dot{\zeta} = \varphi(\zeta, x, v, p^0)$ and $u = \psi(\zeta, x, v, p^0)$, where ζ is the k-dimensional state vector of the feedback and v is the m-dimensional new input.

The resulting closed-loop system with the nominal plant $(p = p^0)$ is m decoupled chains of integrators $y_i^{(\mu_i)} = v_i$ with $i = 1, \dots m$ and $n + k = \sum_{i=1}^{m} \mu_i$. Recall also that it follows from feedback linearization that the time derivatives of the linearizing outputs (outputs of the integrators in the chains) can be expressed as functions of x and ζ: $y_i^{(j)} = h_{i,j}(x, \zeta, p^0)$ for $j = 1, \dots, \mu_i - 1$. The linearized dynamics (integrator chain) can be used to guarantee the exponential decay of tracking error for any sufficiently smooth reference trajectory, denoted as $\bar{y}_r$. The tracking feedback expression can be readily obtained by setting a linear differential equation for each component of the tracking error $e = \bar{y}_r - y$ as $0 = e_i^{(\mu_i)} + \lambda_{i,1} e_i^{(\mu_i - 1)} + \lambda_{i,2} e_i^{(\mu_i - 2)} + \dots + \lambda_{i,\mu_1} e_i$ so that the associated characteristic polynomial is Hurwitz. It follows from the above that the closed-loop transfer between $\bar{y}_r$ and y, assuming $p = p^0$, reads $Y(s) = \text{diag}_{i=1,\dots,m} \left\{ \frac{\lambda_{i,\mu_i}}{s^{\mu_i} + \lambda_{i,1} s^{\mu_i - 1} + \dots + \lambda_{i,\mu_i}} \right\} \bar{Y}_r(s) = G_0(s) \bar{Y}_r(s)$.

However, for non-nominal values of the uncertainty parameters, the closed-loop dynamics is different of $G_0(s)$. Therefore, a finite grid is chosen to cover the uncertainty range Q. For each vertex p^i of the grid, a linear transfer matrix $G^i(s)$ is obtained between the signals $\bar{y}_r$ and y by linearization. This set is covered by an output multiplicative structure with weighting matrices $W_1(s)$ and $W_2(s)$ so that $G^i(s) = (1 + W_1(s) \Delta^i(s) W_2(s)) G_0(s)$ holds true for some $\left\| \Delta^i(s) \right\|_\infty \leq 1$. The weighting functions are numerically determined using Matlab. The associated mixed sensitivity $\mathcal{H}_\infty$ design problem is also solved with the help of Matlab and an augmented plant is used to design the pre-filter $K(s)$ with weighting transfer functions $M(s)$, $W_e(s)$ and $W_u(s)$, denoting the desired closed loop model, the error wright and the input weight transfer functions respectively.

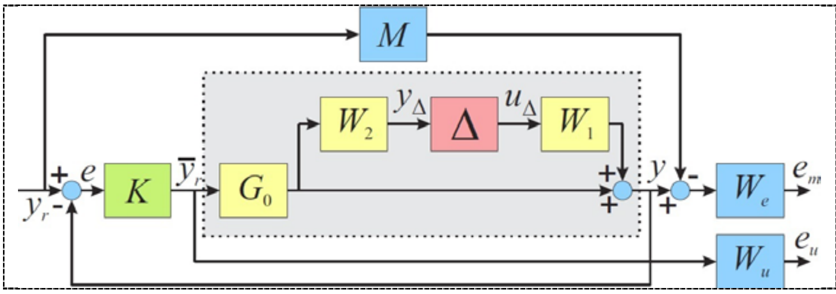

Figure 1 . The controller $K(s)$ with the uncertainty structure and the performance weighting functions.

3. SCARA Robot Control Architecture, Synthesis and Implementation

In order to demonstrate the feasibility of the proposed method, the design procedure described in the previous section is applied to a SCARA type robot (Bosch Turboscara SR 60) [9] as shown in Figure 3.

The SCARA robot has three rotary joints, whose axes are parallel with each other, and a prismatic joint. Using the Euler-Lagrange formalism, the dynamics of robot manipulators with rigid links can be written as $H(q)\ddot{q} + C(q, \dot{q})\dot{q} + B(\dot{q}) + G(q) = \tau$ where q is the vector of joint variables, $H(q)$ is the positive definite inertia matrix, $C(q,\dot{q})$ is the matrix of Coriolis and centripetal forces, $B(\dot{q})$ is the vector or joint friction forces, $G(q)$ is the vector of gravitational forces and τ is the vector of actuator forces. It our case, the dynamic equation reads

$$\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\\ \tau_4\end{bmatrix}=\begin{bmatrix}D_{11} & D_{12} & 0 & D_{14}\\ D_{21} & D_{22} & 0 & D_{24}\\ 0 & 0 & D_{33} & 0\\ D_{41} & D_{42} & 0 & D_{44}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\\ \ddot{q}_3\\ \ddot{q}_4\end{bmatrix}+\begin{bmatrix}D_{112}\dot{q}_1\dot{q}_2+D_{122}\dot{q}_2^2\\ D_{211}\dot{q}_1^2\\ -(m_3+m_4)g\\ 0\end{bmatrix}+\begin{bmatrix}F_{v1}\dot{q}_1\\ F_{v2}\dot{q}_2\\ F_{v3}\dot{q}_3\\ F_{v4}\dot{q}_4\end{bmatrix}$$

where the expressions of the inertia parameters are readily available in the literature.

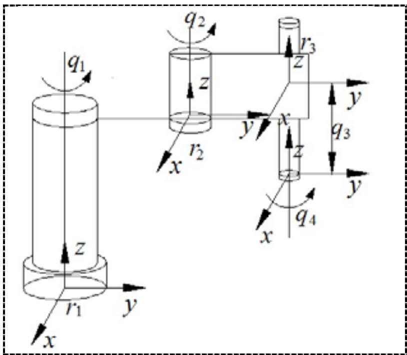

Figure 2 . SCARA coordinate systems and joint variables for RRTR joint configuration.

In most applications, the load mass can only be estimated with limited accuracy. In our case, m_i denote the segment masses. Values of m_1, m_2 and m_3 are supposed to be known with sufficient accuracy: 15kg, 12kg and 3kg, respectively. The payload mass m_4 remains variable. The lengths of the two first segments are also known $l_1 = 0.5m$ and $l_2 = 0.4m$. The terms F_{vi} denote the friction coefficients of the joint axes. The uncertain parameters are enumerated in Table 1.

Table 1. Uncertain parameters of the SCARA

Variable	Nominal value	Minimum value	Maximum value
m_4	3 kg	2 kg	4 kg
F_{v1}, F_{v2}, F_{v4}	0.5 kgm/s	0.2 kgm/s	0.8 kgm/s
F_{v3}	0.05 kgm/s	0.02 kgm/s	0.08 kgm/s

The design procedure suggested in the previous section is followed. The five-dimensional parameter space is covered by a grid and the a linearized dynamics is determined for each vertex. The resulting set is represented by an uncertainty structure so that the `ucover` function from the Robust Control Toolbox of Matlab is used to compute the bounding diagonal weight transfer matrices $W_1(s)$ and $W_2(s)$.

$$W_{1,11}(s)=\frac{0.2684s^2+0.00963s+2.541\cdot10^{-5}}{s^2+2.097s+0.03992} \qquad W_{1,22}(s)=\frac{0.3504s^2+0.6235s+0.001139}{s^2+3.459s+3.105}$$
$$W_{1,33}(s)=\frac{0.4894s^2+0.3171s+0.007531}{s^2+3.108s+3.82} \qquad W_{1,44}(s)=\frac{1.154s^2+0.3297s+0.001853}{s^2+3.905s+1.819}$$
$$W_{2,11}(s)=\frac{0.7388s^2+0.3062s+0.002246}{s^2+3.799s+1.828} \qquad W_{2,22}(s)=\frac{0.7177s^2+8.693s+0.01301}{s^2+15.45s+32.65}$$
$$W_{2,33}(s)=\frac{0.4894s^2+0.3171s+0.007531}{s^2+3.108s+3.82} \qquad W_{2,44}(s)=\frac{0.755s^2+0.3884s+0.006696}{s^2+3.502s+3.632}$$

A reference model $M(s)$ is used to define the desired closed-loop behaviour after the application of the robustifying pre-filter. It gives the transfer between the joint reference signal y_r and the real joint angle signal y. Recall that $\bar{y}_r$ becomes the output of the robustifying pre-filter. The desired closed-loop behaviour is second order and reads

$$M(s)=\mathrm{diag}\left\{\frac{\omega_m^2}{s^2+2\xi_m\omega_m s+\omega_m^2}\right\}$$

with $\omega_m = 2\text{rad/s}$ and $\xi = 0.8$. The performance weighting transfer matrices (see Figure 1) are also chosen to be diagonal (4-by-4). The weighting function are set to

$$W_u(s) = \text{diag}\left\{\frac{40(s+10\omega_m)^2}{(s+200\omega_m)^2}; \frac{40(s+10\omega_m)^2}{(s+200\omega_m)^2}; \frac{40(s+10\omega_m)^2}{(s+200\omega_m)^2}; \frac{1}{5}\frac{40(s+10\omega_m)^2}{(s+200\omega_m)^2}\right\}$$

$$W_e(s) = \text{diag}\left\{\frac{1000\omega_m}{s+2\omega_m}; \frac{1000\omega_m}{s+2\omega_m}; \frac{9}{20}\frac{1000\omega_m}{s+2\omega_m}; \frac{19}{40}\frac{1000\omega_m}{s+2\omega_m}\right\}$$

The robot dynamics and the controller are simulated for different parameter values using Matlab and Simulink. The reference trajectory is set to $y_r = \sin(t)$. The state vector of the robot dynamics is $x = [q, \dot{q}]$. The initial conditions are set accordingly to $x(0) = [0,0,0,0,1,1,1,0]^T$.

Figure 3 shows the tracking performance without the use of the pre-filter whereas Figure 4 shows the joint trajectories with the pre-filter. For each joint, a family of curves are presented, each for a different vertex of the grid spanned over the parameter space. It can be observed that the use of the pre-filter reduces the effect of uncertainty.

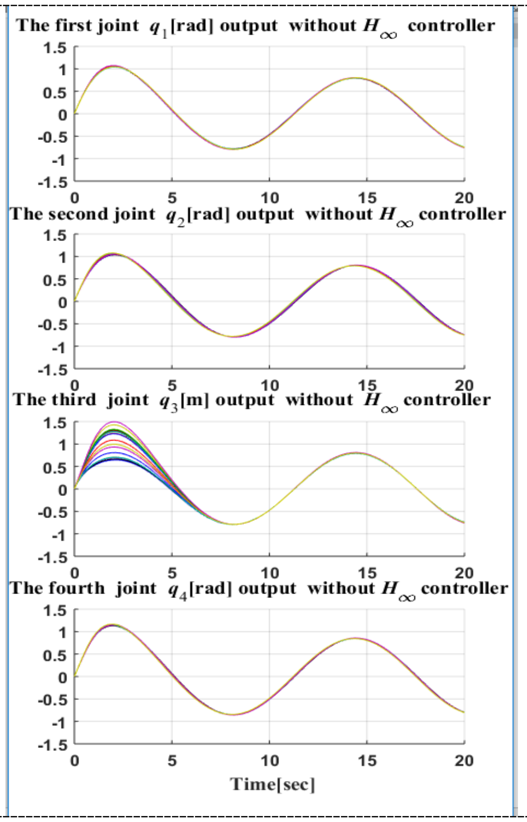

Figure 3. Joint trajectories without $\mathcal{H}_\infty$ pre-filter

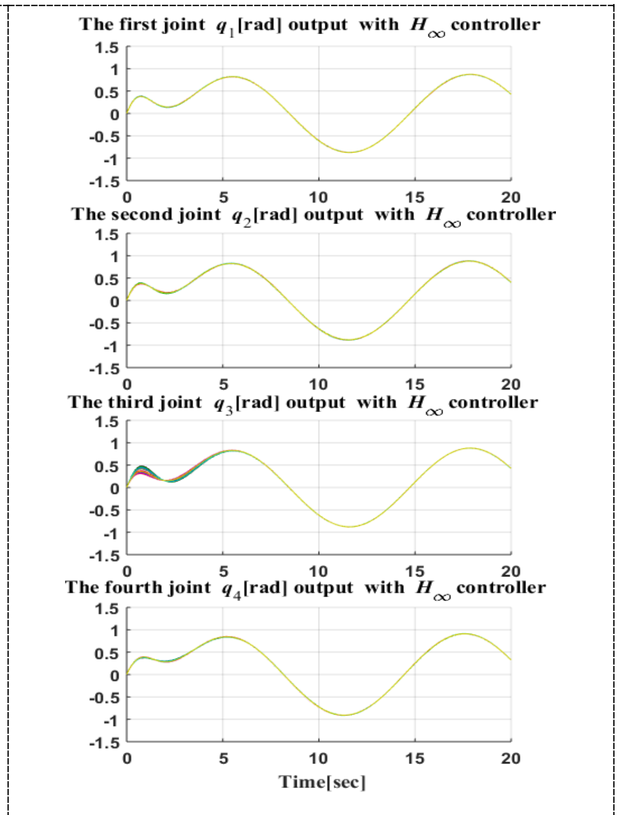

Figure 4. Joint trajectories with the application of $\mathcal{H}_\infty$ pre-filter

4. References

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